Lesson 16 - Solids of Revolution - Washers, Part II

I. Other Axes

II. Examples

Announcement: Bring template for Lessons 17 & 18 on Mon & Wed.

I. Other Axes

Today: Rotate around vertical and horizontal axes that are not the x-axis or y-axis.

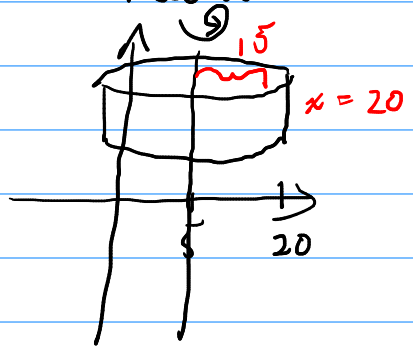
Key ideas

① $x = k$ vertical line like the y-axis ($x = 0$)

$y = k$ horizontal line like the x-axis ($y = 0$)

② Finding the radii

Ex:



distance on a # line: big - small

Horizontal # line: right - left

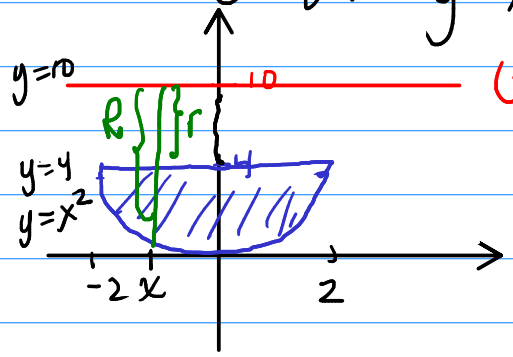
Vertical # line: top - bottom

II. Examples

Ex: Find the volume of the solid obtained by rotating the region $\mathcal{R}$ about the given axis.

(a) $\mathcal{R}$: bounded by $y = x^2$, $y = 4$

axis: $y = 10$



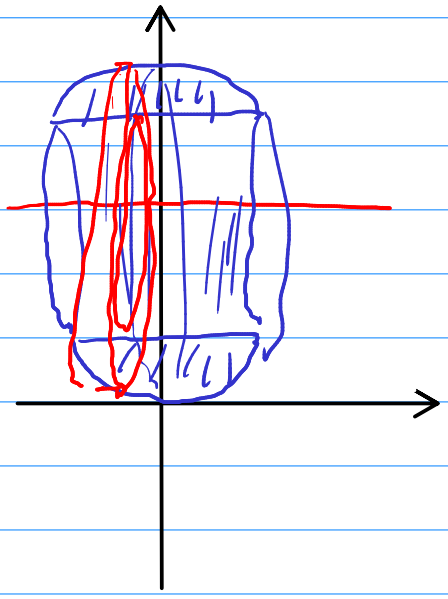
$y = 10$ horizontal like x-axis

$\Rightarrow$ use dx

intersection pts

$x^2 = y$

$x = \pm 2$



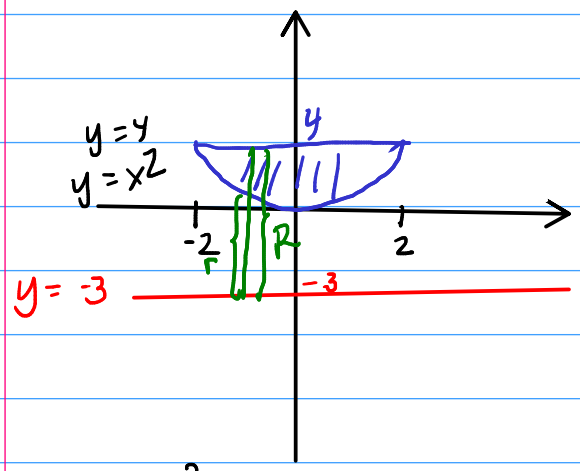
$$V = \int_{-2}^{2} \pi \left(\underbrace{(10 - x^2)^2}_{R^2} - \underbrace{(10 - 4)^2}_{r^2} \right) dx$$

$$= \pi \int_{-2}^{2} \left(100 - 20x^2 + x^4 - 36 \right) dx$$

$$= \pi \int_{-2}^{2} \left(64 - 20x^2 + x^4 \right) dx$$

$$= \pi \left[64x - \frac{20x^3}{3} + \frac{x^5}{5} \Big/_{-2}^{2} \right] = \frac{2432}{15} \pi$$

(b) $\mathcal{R}$: bounded by $y = x^2$, $y = 4$
axis: $y = -3$



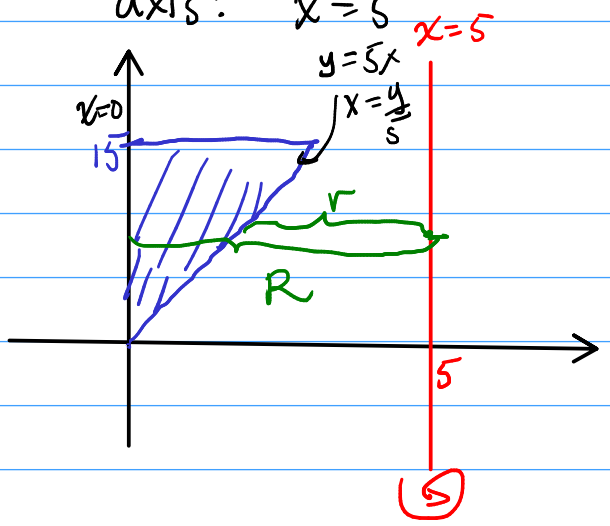
(b) horizontal
like x-axis
use dx

$$V = \int_{-2}^{2} \pi \left(\underbrace{(4-(-3))}_{R^2}{}^2 - \underbrace{(x^2-(-3))}_{r^2}{}^2 \right) dx$$

$$= \int_{-2}^{2} \pi \left(7^2 - (x^2+3)^2 \right) dx \qquad = \frac{576}{5}\pi$$

c) R: bounded by $y = 5x$, $y = 15$, $x = 0$ (y-axis)

axis: $x = 5$



$x = 5$ is a vertical axis like the y-axis $\Rightarrow$ use dy

$$V = \int_0^{15} \pi \left(\underbrace{(5-0)^2}_{R^2} - \underbrace{\left(5-\frac{y}{5}\right)^2}_{r^2} \right) dy$$

$$= \pi \int_0^{15} \left(5^2 - \left(25 - 2y + \frac{y^2}{25}\right) \right) dy$$

$$= 180\pi$$